Homework 2 Solutions

Due: Wednesday, February 11

Problem 2.56

From past experiences a stockbroker believes that under present economic conditions a customer will invest in tax-free bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3, and will invest in both tax-free bonds and mutual funds with a probability of 0.15. At this time, find the probability that a customer will invest

(a) in either tax-free bonds or mutual funds but not both;

(b) in neither tax-free bonds nor mutual funds.

Solution: Consider the events

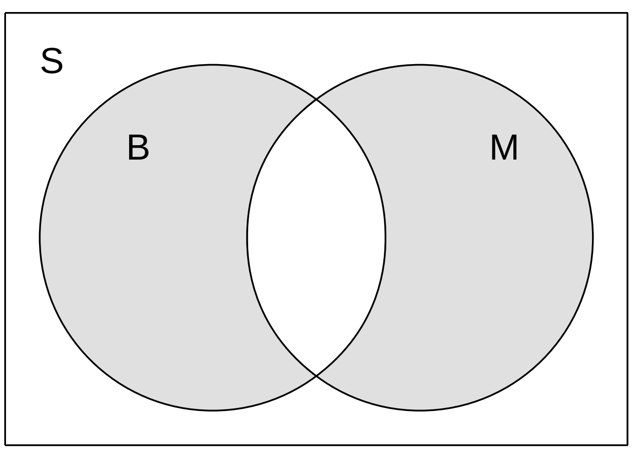
B: customer invests in tax-free bonds,

M: customer invests in mutual funds.

Then:

(a) The event is represented in the Venn diagram below by the shaded area. Its probability

$$\begin{aligned} P\big((B \cap M') \cup (M \cap B')\big) &= P(B \cap M') + P(M \cap B') \\ &= \big(P(B) - P(B \cap M)\big) + \big(P(M) - P(B \cap M)\big) \\ &= P(B) + P(M) - 2P(B \cap M) = 0.6 + 0.3 - 2 \cdot 0.15 = 0.6. \end{aligned}$$



(b) $P(B' \cap M') = 1 - P(B \cup M)$. Since $P(B \cup M) = P(B) + P(M) - P(B \cap M) = 0.6 + 0.3 - 0.15 = 0.75$, we have $P(B' \cap M') = 1 - 0.75 = 0.25$.

Problem 2.60

A pair of fair dice is tossed. Find the probability of getting

(a) a total of 8;

(b) at most a total of 5.

Solution:

(a) The sample space contains $6 \cdot 6 = 36$ equally likely outcomes. The outcomes such that the numbers on the dice add to 8 are:

$$(2,6), (3,5), (4,4), (5,3), (6,2)$$

and there are 5 of them. Hence the probability of obtaining a total of 8 is then $5/36$.

(b) The elements of the sample space totalling at most 5 are:

$$(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)$$

There are ten of them, hence the probability of obtaining at most 5 is 10/36=5/18.

Problem 2.78

A class in advanced physics is comprised of 10 juniors, 30 seniors, and 10 graduate students. The final grades show that 3 of the juniors, 10 of the seniors, and 5 of the graduate students received an A for the course. If a student is chosen at random from this class and is found to have earned an A, what is the probability that he or she is a senior?

Solution: Let N stand for the event that the student is a senior, and A for the event that he or she earned an A. Then

$$P(A \cap N) = 10/50 \qquad P(A) = \frac{3 + 10 + 5}{50} = 18/50$$

and

$$P(N|A) = \frac{P(A \cap N)}{P(A)} = \frac{10/50}{18/50} = 10/18 = 5/9.$$

We could get the same immediately as

$$P(N|A) = \frac{10}{3 + 10 + 5} = 10/18 = 5/9.$$

Problem 2.86

For married couples living in a certain suburb, the probability that the husband will vote on a bond referendum is 0.21, the probability that his wife will vote in the referendum is 0.28, and the probability that both the husband and wife will vote is 0.15. What is the probability that

(a) at least one member of a married couple will vote?

(b) a wife will vote, given that her husband will vote?

(c) a husband will vote, given that his wife does not vote?

Solution: Consider the events:

H: the husband will vote on the bond referendum,

W: the wife will vote on the bond referendum.

Then $P(H) = 0.21$, $P(W) = 0.28$, and $P(H \cap W) = 0.15$.

(a) $P(H \cup W) = P(H) + P(W) - P(H \cap W) = 0.21 + 0.28 - 0.15 = 0.34$.

(b) $P(W|H) = \frac{P(H \cap W)}{P(H)} = \frac{0.15}{0.21} = \frac{5}{7}$.

(c) $P(H|W') = \frac{P(H \cap W')}{P(W')} = \frac{P(H) - P(H \cap W)}{1 - P(W)} = \frac{0.06}{0.72} = \frac{1}{12}$.

Problem 2.88

The probability that the head of a household is home when a telemarketing representative calls is 0.5. Given that the head of the house is home, the probability that goods will be bought from the company is 0.3. Find the probability that the head of the house is home and goods being bought from the company.

Solution: Define events

H: head of household is home,

G: goods are bought from the company.

Then

$$P(H \cap G) = P(H)P(G|H) = 0.5 \cdot 0.3 = 0.15$$

Problem 2.82

A manufacturer of flu vaccine is concerned about the quality of its flu serum. Batches of serum are processed by three different departments having rejection rates of 0.10, 0.08, and 0.12, respectively. The inspections by the three departments are sequential and independent.

(a) What is the probability that a batch of serum survives the first departmental inspection but is rejected by the second department?

(b) What is the probability that a batch of serum is rejected by the third department?

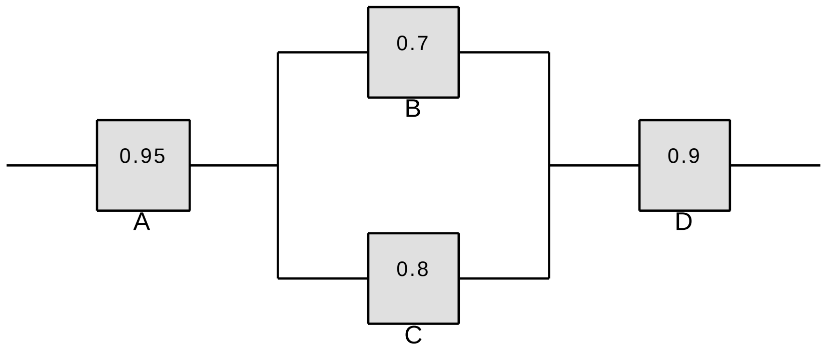
Solution: Let D_1, D_2, D_3 be the events that the serum would be rejected by departments 1, 2, 3 respectively. Then $P(D_1) = 0.10$, $P(D_2) = 0.08$, $P(D_3) = 0.12$, and all three events are independent.

(a) $P(D_1' \cap D_2) = \big(1 - P(D_1)\big)P(D_2) = 0.90 \cdot 0.08 = 0.072$.

(b) $P(D_1' \cap D_2' \cap D_3) = \big(1 - P(D_1)\big)\big(1 - P(D_2)\big)P(D_3) = 0.90 \cdot 0.92 \cdot 0.12 \approx 0.099$.

Problem 2.98 (bonus)

Suppose the diagram of an electrical system is given in the figure below. What is the probability that the system works? Assume the components fail independently.



Solution: Let A, B, C, and D denote the events that the respective components work. Then

$$P(A) = 0.95 \qquad P(B) = 0.7 \qquad P(C) = 0.8 \qquad P(D) = 0.9$$

and all the events are independent. The system works (event W) if both components A and D work, and at least one of the components B or C works. That is $W = A \cap (B \cup C) \cap D$ and

$$\begin{aligned} P(W) &= P\big(A \cap (B \cup C) \cap D\big) \\ &= P(A)P(B \cup C)P(D) \\ &= P(A)\big(1 - P(B' \cap C')\big)P(D) \\ &= P(A)\big(1 - P(B')P(C')\big)P(D) \\ &= 0.95(1 - 0.3 \cdot 0.2)0.9 \approx 0.8037 \end{aligned}$$